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Large Enhancement of Heavy-Ion Fusion Cross section at Energies around the Coulomb Barrier

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Abstract

The purpose of this research paper is to review some basics of theoretical approaches to Heavy-ion fusion reactions at energies around the Coulomb barrier. In the parabolic approximation of a barrier the transmission coefficients are given by the Hill-Wheeler expression and the cross section is given by the Wong formula; this is the so-called uncoupled fusion. The one dimensional model can describe the fusion of light systems like $^{14}N+^{12}C$, $^{12}C+^{29}Si$ and $^{16}O+^{27}Al$. However, it fails to reproduce the experimental data of the excitation function of the fusion cross section for heavy systems such as $^{40}Ca+^{62}Ni$, $^{40}Ar+^{144}Sm$ and $^{16}O+^{154,148,144}Sm$. Large enhancements of fusion cross section against predictions of the potential model can be caused by the coupling of the relative motion between the colliding nuclei to other degrees of freedom, e.g. their intrinsic excitations, nuclear transfer and etc. They are called channel-coupling effects. These effects can be effectively expressed in terms of distribution of potential barriers.

Keywords: Heavy-ion subbarrier fusion reaction; Penetrability; Nucleus-nucleus potential; Fusion barrier distribution

1. Introduction

Fusion is defined as a reaction where two separate nuclei combine together to form a composite system. In this contribution, we deal with fusion reactions at energies near and below the Coulomb barrier, where heavy-ion fusion reactions are governed by quantum tunneling.

In the study of fusion reactions below the Coulomb barrier created by the strong cancellation between the long-range repulsive Coulomb force and the short-range attractive nuclear interaction, the experimental observables are the cross section and the average angular momentum. In general, the compound nucleus formed in heavy-ion fusion reaction is highly excited and decays either by emitting neutrons, protons, α particles, γ and X rays (which are called particle evaporation), or by fission. The dominant decay mode of the compound nucleus is particle evaporations for lighter systems ($Z < 70$, Z being the atomic number) and fission for heavy systems ($Z > 90$). Experimentally, the total fusion cross section is then defined by the

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sum of the fission and the evaporation residue cross sections. Therefore, fusion cross sections at low energies are measured by detecting evaporation residues or fission products from compound nucleus formation. A review of different experimental techniques for measuring the fusion cross sections is given by Beckerman (1988). It is worthwhile to emphasize that moments of the angular momentum distributions, unlike the fusion cross section itself, are not directly measurable quantities [3, 1].

Theoretically, it has been considered that fusion has been achieved once colliding nuclei overcome the Coulomb barrier because of strong absorption by nuclear force inside the Coulomb barrier if the charge product of the system is less than 1,600.

The fusion cross section at energy E is equivalent to the transmission cross section of the Coulomb barrier given by the standard formula:

$$\sigma_F(E) = \frac{\pi\hbar^2}{2\mu E}\sum_l (2l+1)\, P_l(E), \tag{1}$$

where μ is the reduced mass of the system and $P_l(E)$ is the barrier transmission probability for an angular momentum l. Now the important thing in the study of fusion reactions is how to precisely estimate the barrier transmission probability.

In 1973, Wong derived the analytical expressions (1) of fusion cross sections if the Coulomb barrier is approximated by a parabolic form [2], which is generally called Wong formula. The experiment data obtained by Vax, Alexander and Satcher(1981)[3] showed that the Wong formula successfully reproduces the observed cross sections for fusion of light ions but it fails to reproduce those for heavy ions, which showed huge enhancement at energies below the Coulomb barrier. This indicates that the Wong formula, which relies on the assumption that fusion reaction can be described by the relative motion of colliding nuclei and the Coulomb barrier can be approximated by a parabolic form, are no longer valid.

In the next section, we describe the one-dimensional potential model which estimates the fusion cross section in terms of the penetrability (or the fusion probability) for the l-wave scattering and approximate this penetrability by the uniform WKB approximation. In section (3), the simplest potential model with parabolic approximation and Wong formula is derived. In section (4), its failures in reproducing the observed fusion cross sections for medium weight mass systems, and the reasons why a one-dimensional description fails, are demonstrated.

In section (5), we present that the effects of channel coupling can be expressed in terms of distribution of potential barriers. In this section we discuss the transparent behavior of channel coupling by means of a schematic two-channel problem with the help of the Wong formula. Finally sect. (6) summaries this contribution.

2. One Dimensional Potential Model

Theoretically, the simplest approach to heavy-ion fusion reaction is to use the one-dimensional potential model where the reaction is described only by the relative distance r between the projectile and target and both the projectile and the target are assumed to be structure less. One of the basic concepts of the nuclear reactions is nucleus-nucleus potential that is a function of the distance r between the centre of mass of the target and projectile. It consists of a repulsive Coulomb term $V_C(r)$ and a short ranged attractive nuclear term $V_N(r)$. The nucleus-nucleus potential $V_0(r)$ can be written as the sum of the Coulomb and nuclear potential and centrifugal potential,

$$V(r) = V_N(r) + V_C(r) + V_l(r). \tag{2}$$

Fig. (1) shows a potential $V_0(r)$ for the s-wave scattering of the ^{16}O + ^{144}Sm reaction. The thicker and the thinner lines are the Coulomb and the nuclear potentials, while the total potential $V_0(r)$ is denoted by the dotted line. The nuclear potential is usually taken to be of Wood-Saxon form, $V_N(r) = -\frac{V_0}{1+\exp\left[\frac{r-R_0}{a}\right]}$ which is defined by three parameters: the depth V_0, the radius R_0 and the diffuseness a [4]. At the present time the physical origin of the large value of surface diffuseness parameter in the nuclear potential required to fit data is an open problem [5]. A potential barrier appears to the compensation between the attractive nuclear force and the repulsive Coulomb force and is called the Coulomb barrier. Properties of the Coulomb barrier are characterized by the Coulomb barrier position r_B barrier curvature Ω and barrier height V_B [6]. In describing fusion, the potential around the fusion barrier radius r_B is most important and the presence of a pocket in the nuclear potential allows a simple conceptual criterion for fusion. Once the significant density overlapping occurs, a substantial loss of kinetic energy and angular momentum occurs from the relative motion to nuclear intrinsic degrees of freedom and all the flux passing the barrier lead to fusion [7]. The theory of scattering defines the fusion cross section $\sigma_F(E)$ at an energy E and it can be given by a summation over all partial waves,

$$\sigma_F(E) = \frac{\pi}{k^2} \sum_l (2l+1) P_l(E), \quad (3)$$

where $P_l(E)$ is the penetrability or the fusion probability for the l-wave scattering and is determined by numerically solving the Schrödinger equation for the radial motion. Alternatively, one can approximate by the uniform WKB approximation as

$$P_l(E) = \left\{1 + exp\left[2 \int_{r_2}^{r_1} k(r) dr\right]\right\}^{-1}, \quad (4)$$

where $k(r)$ is the local wave number, and r_2 and r_1 are the inner and outer classical turning points at the potential barrier. Then the parabolic approximation at this barrier will be reviewed in the following section.

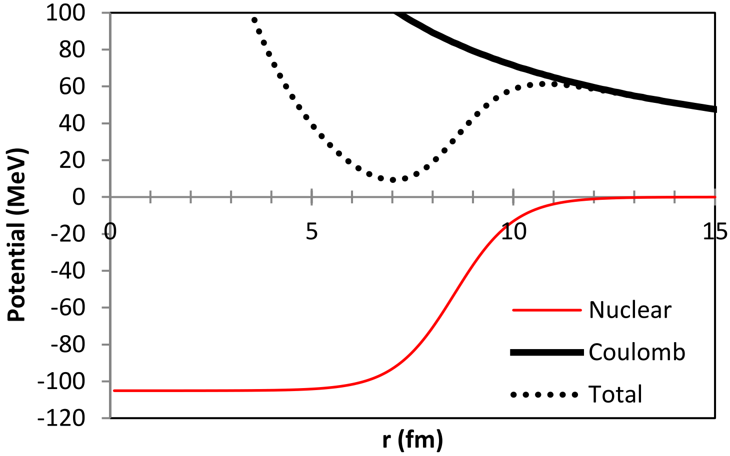


Fig. 1 ; A typical potential $V_0(r)$ for the s-wave scattering of the ^{16}O + ^{144}Sm reaction as a function of the relative distance between ^{16}O + ^{144}Sm.

3. Parabolic Approximation and Wong Formula

As was reviewed in the former section, near the barrier top any reasonable potential can be approximated by a parabola

$$V_0(r) = V_B - \frac{1}{2}\mu^2 \Omega^2 (r - r_B)^2 . \quad (5)$$

Then the penetrability

$$P_l(E) = \left\{1 + exp\left[\frac{2\pi}{\hbar\Omega_l}(V_B - E)\right]\right\}^{-1} , \quad (6)$$

can be analytically evaluated in Eq. (3). In the nuclear physics literature, Eq. (6) is known as the Hill-Wheeler formula. V_B and Ω_l are the height and the curvature of the fusion barrier for the partial wave l, respectively. In Fig. (2) the Coulomb barrier for the s-wave scattering of $^{16}O+^{144}Sm$ reaction is compared with the parabolic potential. Akyüz-Winther potential is used for the nuclear potential. The curvature, barrier height and barrier position of this potential are 4.25 MeV, 61.24 MeV and 10.81 fm respectively. Because of the long ranged Coulomb interaction, the parabolic potential has less width compared with the realistic situation. Fig. (3) compares the penetrability of the s-wave scattering obtained by numerically solving the Schr ö dinger equation with that obtained in the parabolic approximation. Although the parabolic approximation overestimates the agreement between the exact solution and the approximation is remarkable, especially at energies above the Coulomb barrier. Using the parabolic approximation, Wong has derived an analytic expression of fusion cross sections. He assumed that (i) the curvature of the Coulomb barrier is independent of the angular momentum l, and (ii) the dependence of the penetrability on the angular momentum can be well approximated by the shift of the incident energy as

$$P_l(E) = P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu r_B^2}\right). \quad (7)$$

If many partial waves contributed to fusion cross section, the sum in Eq. (3) can be replaced by an integral;

$$\sigma_F(E) = \frac{\pi}{k^2}\int_0^\infty dl(2l+1)\,P_l(E). \quad (8)$$

Ignoring the l dependence of curvature Ω and barrier position r_B, one can obtain following Wong formula;

$$\sigma(E) = \frac{\hbar\Omega}{2E} r_B^2 Log\left[1 + exp\left(\frac{2\pi}{\hbar\Omega}(E - V_B)\right)\right]. \quad (9)$$

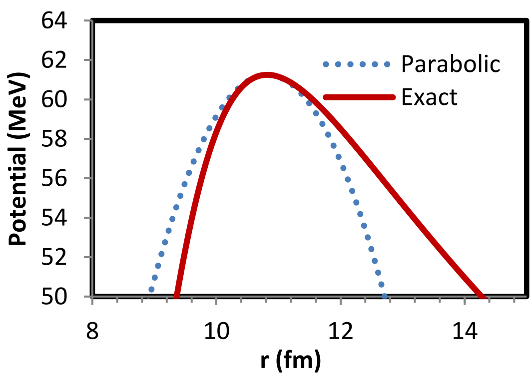


Fig. 2 ; Comparison between the Coulomb barrier for the $^{16}O+^{144}Sm$ reaction (the solid line) and a parabolic potential (the dotted line).

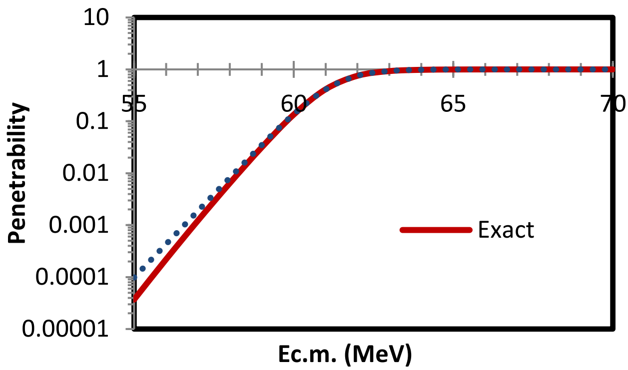


Fig. 3 ; The validity of the parabolic approximation for the s-wave penetrability of the ^{16}O + ^{144}Sm reaction.

parameters; the barrier height V_B , the barrier radius r_B and the curvature Ω_B . At high energies above the Coulomb barrier, the exponential in the argument of the logarithm in expression (9) is much larger than unity. This formula gives the classical expressions,

$$\sigma(E) = \pi r_B^2 \left(1 - \frac{V_B}{E}\right) \text{ for } E > V_B \ , \tag{10}$$

while at low energies, the exponential term is small, then

$$\sigma(E) \approx r_B^2 \frac{\hbar\Omega}{2E} exp\left(\frac{2\pi}{\hbar\Omega}(E - V_B)\right) \text{ for } E < V_B \ . \tag{11}$$

Fig. (4) shows the comparison of fusion cross section for the ^{16}O+^{144}Sm reaction obtained by the Wong formula with exact numerical solutions. One can observe that the Wong formula works very well except below the Coulomb barrier where the parabolic approximation break down.

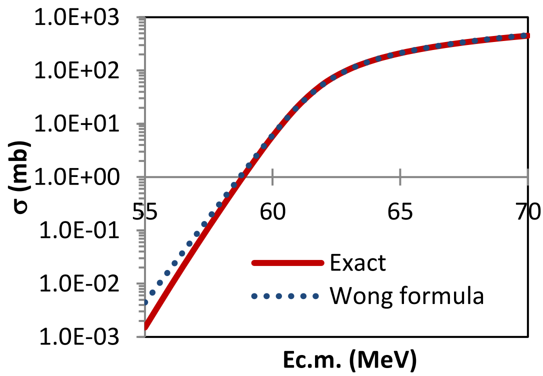


Fig. 4 ; The comparison of fusion excitation functions of the ^{16}O+^{144}Sm reaction.

4. Comparison with Experimental Data; Failure of the Potential Model

We now review the comparison of experimental excitation functions of fusion cross section for several systems with predictions of the one dimensional potential model (Wong formula). In Fig. (5) taken from Vaz et al. [1], the solid lines are predictions of the potential model obtained by using the parabolic approximation. One can find that the potential model reproduces the experimental data very well for relatively light systems, i.e. the $^{14}N+^{12}C$, $^{16}O+^{27}Al$ and $^{12}C+^{29}Si$ reactions. However, this is no longer so in the case of heavier systems where the cross section at sub barrier energies are significantly larger than the one dimensional model. Fig. (6) shows the experimental fusion excitation function for the $^{16}O+^{144,148,154}Sm$ reactions and comparisons with the potential model (the solid lines). These are plotted as functions of the difference between the centre of mass energy and the barrier height for each reaction. The barrier height and the result of the potential model are obtained by using the Akyüz-Winther potential [8]. We again observe that the experimental fusion cross sections drastically enhance compared with the predictions of the potential model. Moreover, we observe that the degree of enhancement of fusion cross section depends strongly on the target nucleus. The enhancement for the $^{16}O+^{154}Sm$ system is order of magnitude while that for the $^{16}O+^{144}Sm$ system is about factor four at energies below the Coulomb barrier.

These discrepancies were not due to the use of wrong of potential rather the assumption of the one dimensional model is not adequate for heavier systems. The inadequacy of the potential model was demonstrated in the inversion formula of experimental data applied by Balantekin et al., where the shape of the potential barrier is obtained by the penetrability deduced from experimental fusion cross sections [3]. They show that there are the limitations of the simple model. A more microscopic description would need to understand the physical effect. The reason for this failure is that an increasing number of inelastic channels have to be taken into account for heavier systems. This will be reviewed in the following.

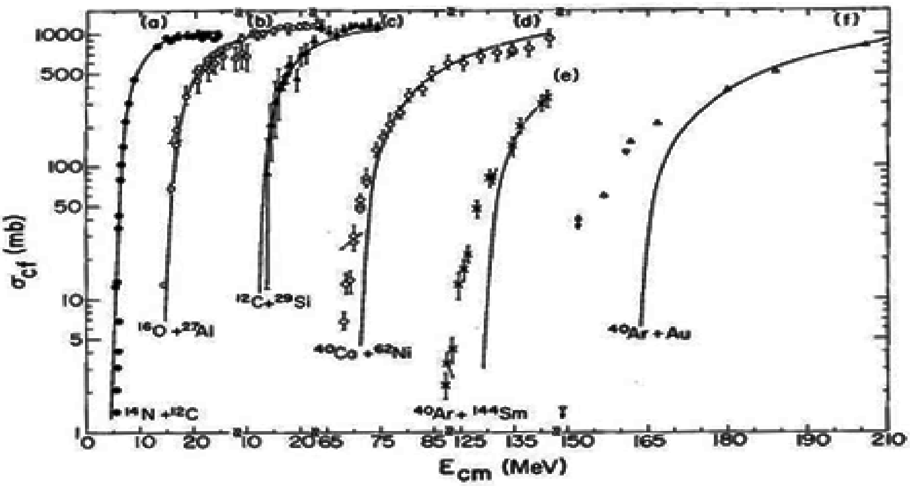


Fig. 5 ; Comparison of experimental excitation functions of fusion cross section for several systems with predictions of the potential model (the solid lines).

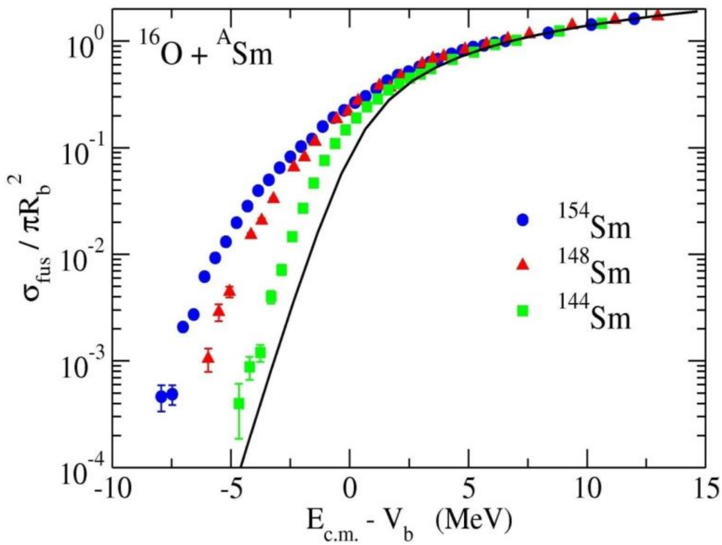


Fig. 6 ; The experimental fusion excitation function for $^{16}O+^{144,148,154}Sm$ reactions.

5. Coupled-Channel Effects and Barrier Distribution Concepts

Extensive experimental as well as theoretical studies have revealed that the inadequacy of the potential model i.e. large enhancements of fusion cross section against predictions of the potential model can be caused by the coupling of the relative motion between the colliding nuclei to other degrees of freedom, e.g. their intrinsic excitations, nuclear transfer. They are called channel-coupling effects [9]. As two fusing nuclei approach each other, they will in general undergo transitions from the ground states to excited states and transfer particles between themselves, before coming close enough to form a compound nucleus. The elastic channel couples to inelastic channels and the transmissions across the barrier takes place in each of these channels. The incoming wave splits up into various inelastic waves with different transmission probabilities. In order to obtain the total transmission into the interior of the compound system, these different transmission probabilities have to be combined. On the other hand, the interactions which are responsible for the coupling also contribute to the effective barriers in the various channels. This leads to an enhancement of the transmission coefficient [2].

Many people in this field are usually solving the Couple-Channels calculation which is a standard theoretical approach to describe heavy-ion fusion reaction by taking the effects of nuclear intrinsic degree of freedom into account. In the earlier fusion, simplified Couple-Channel codes such as CCFUS, CCDEF and CCMOD were widely used. Currently newer codes are available to experimentalists, such as CCFULL.

The large enhancement of the fusion cross section, and also the strong isotope dependence, are caused by the coupling of the relative motion between the projectile and target to their intrinsic degrees of freedom. The effects of channel coupling can be expressed in terms of the distribution of potential barriers when the excitation energy of the intrinsic motion is zero, and the underlying structure of the barrier distribution can be detected by taking the first derivative of penetrability. For a completely classical system, we can see in Fig. (7.a) that P_0 penetrability is unity above the barrier and zero below; hence $\mathrm{d}P_0/\mathrm{d}\,E$ is a delta function peaked when E is equal to the barrier height. Quantum mechanically this sharp peak is broadened as the transmission probability smoothly changes from zero in Fig. (7.b). Rowley et al. suggested that if many channels are coupled to the relative motion, the quantity $\mathrm{d}P_0/\mathrm{d}\,E$ is further broadened and can be taken to represent the “distribution of the barriers”.

In the problem of heavy-ion fusion reaction, the experimental observable is not penetrability, but fusion cross section, and thus if one intend to discuss the effects of channel-coupling on fusion in terms of the first derivative of penetrability, one has to convert fusion cross sections to penetrability's of the s-wave scattering. The Wong formula given by Eq. (9) suggests one prescription for this, i.e. it suggests that the first derivative of the product of fusion cross section σ_f and the centre of mass energy E with respect to the energy, d(Eσ)/d E , is proportional to the penetrability of the s-wave scattering

$$\frac{d(E\sigma)}{dE} = \frac{\pi r_B^2}{1+exp\left[-\frac{2\pi}{\hbar\Omega}(E-V_B)\right]} = \pi r_B^2 P_0(E). \quad (12)$$

This equation immediately leads to a relation between the first derivative of the penetrability and the fusion cross section

$$\frac{d^2(E\sigma)}{dE^2} = \pi r_B^2 \frac{2\pi}{\hbar\Omega} \frac{exp\left[\frac{2\pi}{\hbar\Omega}(E-V_B)\right]}{\left\{1+exp\left[\frac{2\pi}{\hbar\Omega}(E-V_B)\right]\right\}} = \pi r_B^2 \frac{dP_0(E)}{dE} \quad . \quad (13)$$

This quantity, which is conventionally called fusion barrier distribution, is peaked at the height of Coulomb barrier for the s-wave scattering V_B.

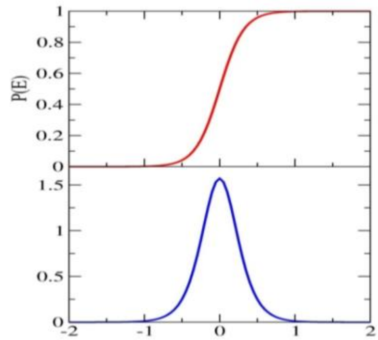


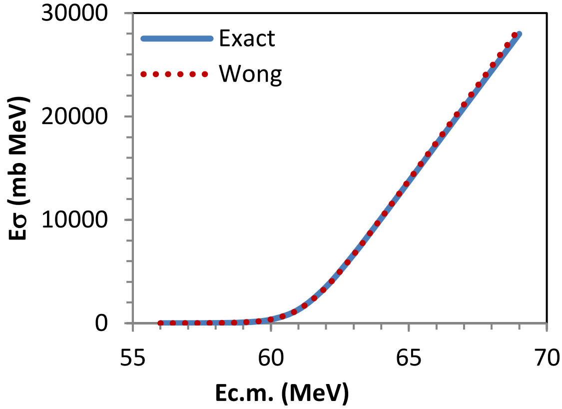
Fig. 7.b ; Quantum mechanically

$$\frac{dP_0}{dE}$$

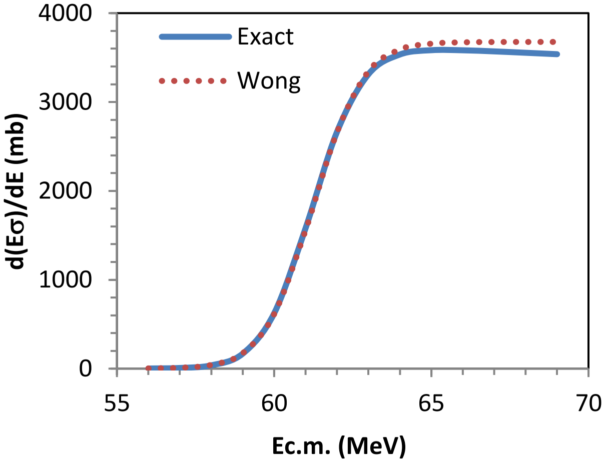
Fig. 7.a ; Classically

Fig. 7 ; Classical (7.a) and quantum mechanical (7.b) transmission probabilities and their first derivatives for a one dimensional potential barrier in typically.

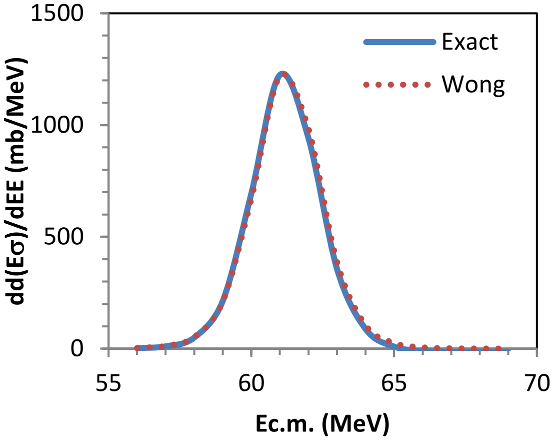
We can also check how the first derivative of $E\sigma_F$ describes the s-wave penetrability as follows; firstly, Fig. (8.a) shows the product of fusion cross section σ_f and the centre of mass energy E with using the Wong formula and without it. Then Fig. (8.b) compares the first derivative d($E\sigma_F$)/dE obtained by numerically solving the Schrödinger equation without using the Wong formula with numerical solution of the s-wave penetrability. The internal excitations of both the projectile and the target nuclei are not taken into account. Although $\mathrm{d}^2 E\sigma_F/\mathrm{d}\,E^2$ decrease at high energies while $\pi r_B^2 P_0$ become close to one, Fig.(8.c) shows a comparison between the second derivative of $E\sigma_f$ and the first derivative of penetrability $\mathrm{d}P_0/\mathrm{d}\,E$ which is scaled by πr_B^2 .



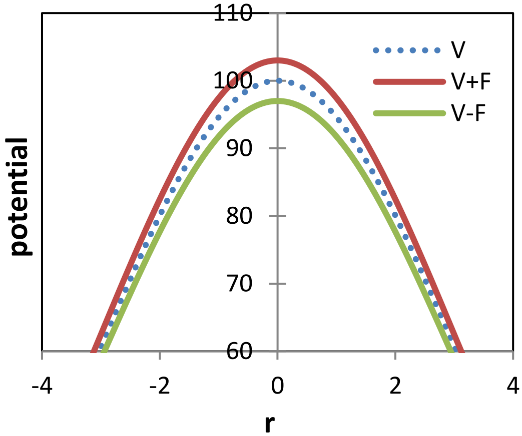
E



B



P_0



The various effects of channel coupling on barrier penetration within a simple model have been illustrated by Dasso et al(1983)[4].The transparent behavior of channel coupling will be studied in a schematic two-channel problem with the help of the Wong formula. Due to the effect of the two channel coupling, the bare potential $V_0(r)$(dotted line) splits into two eigen-barriers $V_0(r)+|F(r)|$(upper solid line) and $V_0(r)$-$|F(r)|$(lower solid line) which are higher and lower than the bare potential in Fig. (9). In this application we use gaussian shape for both the bare potential and the coupling form factor

$$V_0(r) = V_0 e^{-r^2/2s^2} \text{ and } F_0(r) = F_0 e^{-r^2/2s_f^2}, \quad (14)$$

in which V_0=100MeV, F_0=3MeV and $s= s_f$ =3fm, respectively. The penetrabilities of the effective potential V_0 (r)± $F(r)$ are given by

$$P_+ = \frac{1}{1+exp\left[-\frac{2\pi}{\hbar\Omega}(E-V_B+F_0)\right]} \text{ and } P_- = \frac{1}{1+exp\left[-\frac{2\pi}{\hbar\Omega}(E-V_B-F_0)\right]}, \quad (15)$$

where form factor F_0 is assumed about 3MeV. The curvature Ω and the bare potential height $V_B$are chosen in $^{16}O+^{144}Sm$ reaction [1].The total barrier penetrability is given by

$$P(E) = \frac{1}{2}[P_+ + P_-]. \quad (16)$$

Fig. (10) illustrates the penetrability in this model as a function of energy. The solid and dashed curves are calculated with and without coupling between the channels. The penetrability is on the left and fusion barrier distributions are on the right. The $\frac{dP_0(E)}{dE}$ splits into two peaks and the peak positions of $\frac{dP_0(E)}{dE}$ correspond to each barrier height. Because of the channel coupling we can see that the penetrability is decreased at energies above the barrier V_B whereas it is enhanced at energies below V_B. Consequently the fusion cross section is enhanced at energies below the barrier, and reduced above the barrier [2]. These observations explain why the channel coupling enhances the fusion cross section at energies below the original potential barrier. Then the one dimensional potential model have neglected this barrier distribution and assumed only bare potential. So it failed.

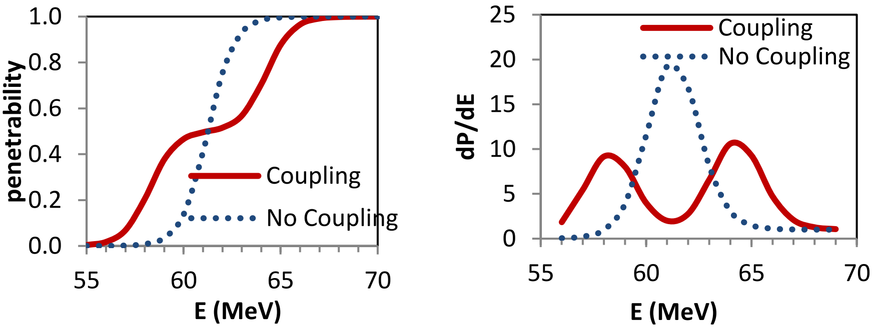


Fig. 10 ; Penetrability functions in the two-channel problem estimated by using Wong formula.

6. Summary

In this contribution we have discussed some aspects of the studies of heavy-ion fusion at energies around the Coulomb barrier. We have shown that the fusion cross section is significantly enhanced compared with the prediction of a one-dimensional potential model and has a strong isotope dependence. Inclusion of coupled channels can take into account distortions of the nuclei due to the strong forces acting on them near the barrier. A very important recent progress is to obtain detailed information of nuclear structure and excitations from fusion data, especially through the so-called fusion barrier distribution analysis. We have demonstrated analytically within the two channel problem that the coupling acts to enhance fusion cross section at energies below the barrier. We have argued that heavy-ion fusion reaction can be a powerful tool to probe details of nuclear deformation and nuclear intrinsic excitation.

Acknowledgement

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